

	Proof of (i)
Let $x \notin K(N, v, S)$ , we v	vant to show that $x \notin Nu(N, v, S)$ .
$x \notin K(N, v, S)$ , hence, the s <sub>lk</sub> (x) > s <sub>kl</sub> (x) and x <sub>k</sub> >	here exists $\mathcal{C} \in CS$ and $(k,l) \in \mathcal{C}^2$ such that $v(\{k\})$ .
Let y be a payoff distr	ribution corresponding to a transfer of utility
$\epsilon > 0$ from k to l: $y_i =$	
Since $x_k > v(\{k\})$ and s enough s.t.	$s_{lk}(x) > s_{kl}(x)$ , we can choose $\epsilon > 0$ small
$\odot \ x_k - \epsilon > v(\{k\})$	
$ \circ \ s_{lk}(y) > s_{kl}(y) $	
We need to show that	$e(y)^{\blacktriangleright} \leq_{lex} e(x)^{\blacktriangleright}.$
Note that for any coal	ition $S \subseteq N$ s.t. $e(S, x) \neq e(S, y)$ we have either
• $k \in S$ and $l \notin S$ (e)	$(S,x) > e(S,y)$ since $e(S,y) = e(S,x) + \epsilon > e(S,x)$
• $k \notin S$ and $l \in S$ (e)	$(S,x) < e(S,y)$ since $e(S,y) = e(S,x) - \epsilon < e(S,x)$

Theorem

Theorem

Proof

 $Jmp \neq \emptyset.$  Then we have: • (i)  $Nu(N,v,S) \subseteq K(N,v,S)$ • (ii)  $K(N,v,S) \subseteq BS(N,v,S)$ 

Properties

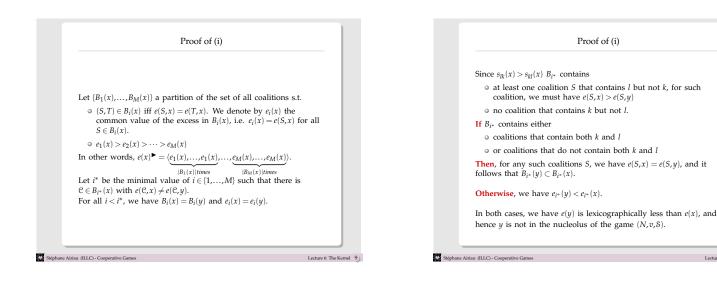
Let (N, v, S) a game with coalition structure, and let

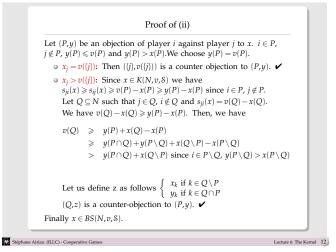
Let (N, v, S) a game with coalition structure, and let  $\Im p \neq \emptyset$ . The kernel K(N, v, S) and the bargaining set

Since the Nucleolus is non-empty when  $\exists mp \neq \emptyset$ , the proof is immediate using the theorem above.

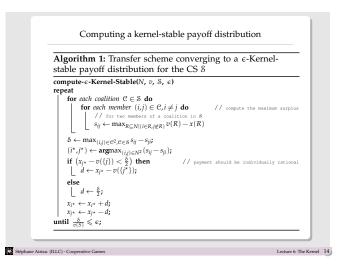
BS(N, v, S) of the game are non-empty.

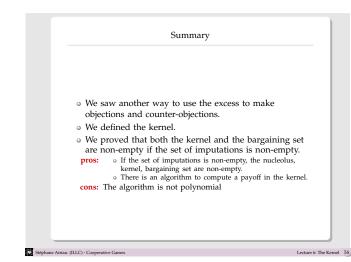
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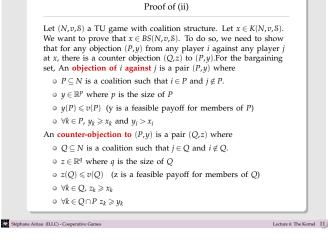


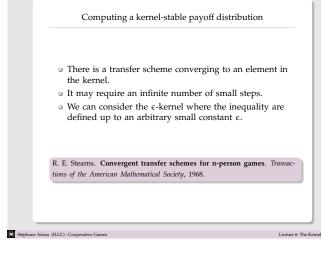


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- The complexity for one side-payment is  $O(n \cdot 2^n)$ .
- Upper bound for the number of iterations for converging to an element of the  $\epsilon$ -kernel:  $n \cdot log_2(\frac{\delta_0}{\epsilon \cdot v(S)})$ , where  $\delta_0$  is the maximum surplus difference in the initial payoff distribution.
- To derive a polynomial algorithm, the number of coalitions must be bounded. For example, only consider coalitions which size is bounded in  $[K_1, K_2]$ . The complexity of the truncated algorithm is  $O(n^2 \cdot n_{coalitions})$ where  $n_{coalitions}$  is the number of coalitions with size in  $[K_1, K_2]$ , which is a polynomial of order  $K_2$ .
- M. Klusch and O. Shehory. A polynomial kernel-oriented coalition algorithm for rational information agents. In Proceedings of the Second International Conference on Multi-Agent Systems, 1996. • O. Shehory and S. Kraus. Feasible formation of coalitions among autonomous agents in non-superadditve environments. Computational Intelligence, 1999.

• The **Shapley value**. It is not a stability concept, but it tries to guarantee fairness. We will see it can be defined axiomatically or using the concept of marginal contributions.

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